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On weak π-regularity of rings whose prime ideals are maximal

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Abstract

We investigate, in this paper, the connections between the weak π-regularity and the maximality of prime ideals in 2-primal rings, right quasi-duo rings and PI-rings, respectively. © 2000 Elsevier Science B.V. All rights reserved.

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The relationship between various generalizations of von Neumann regularity and the condition that every prime ideal is maximal have been investigated by many authors [3, 4, 7, 9, 13, 15, 16]. The first clearly established equivalence between a generalization of von Neumann regularity and the maximality of prime ideals seems to have been made by Storrer [13] in the following result: If R is a commutative ring with identity then R is π-regular if and only if every prime ideal of R is maximal. Storrer's result was extended to PI-rings [7, Theorem 2.3], right duo rings [9, Corollary 1] and bounded weakly right duo rings [15, Theorem 3], respectively. On the other hand, Hirano [9] proved that if R is a 2-primal ring, then R is π-regular if and only if every prime ideal of R is a maximal one-sided ideal. Recently, Birkenmeier et al. [3] showed that if R is a 2-primal ring, then $R/P(R)$ is right weakly π-regular if and only if every prime ideal of R is maximal. These results were mainly explained the relation between the π-regularity and the maximality of prime ideals of rings.

The π-regularity of rings is extended to the weak π-regularity. In general, π-regular rings are weakly π-regular rings but the converse is not hold.

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We investigate the connections between the results of previously mentioned papers and weak π-regularity in 2-primal rings, right quasi-duo rings and PI-rings, respectively. Consequently, our results in this paper extend many of results in [4, 7, 15].

Throughout this paper the letter R denotes an associative ring with identity and all prime ideals of R are assumed to be proper. $P(R)$, $J(R)$ and $N(R)$ denote the prime radical, the Jacobson radical and the set of all nilpotent elements of R, respectively.

We begin with the following definitions.

Definition 1. (1) A ring R is said to be (*strongly*) *π-regular* if for every $x\in R$ there exists a natural number n, depending on x, such that $(x^n\in x^{n+1}R)\,x^n\in x^nRx^n$. Strong π-regularity is right–left symmetric [6].

(2) A ring R is said to be *right* (*left*) *weakly π-regular* if for every $x\in R$ there exists a natural number n, depending on x, such that $x^n\in x^nRx^nR\,(x^n\in Rx^nRx^n)$. R is *weakly π-regular* if it is both right and left weakly π-regular [8].

Definition 2. A ring R is called 2-*primal* if $P(R)=N(R)$ [2].

The term 2-*primal* was come upon originally by Birkenmeier, Heatherly and Lee. But Hirano [9] used the term *N-ring* for what we call a 2-primal ring. The 2-primal condition was taken up independently by Sun [14], where in the setting of rings he introduced a condition to be called *weakly symmetric*, which is equivalent to the 2-primal condition for rings.

Hirano showed the following proposition.

Proposition 3 (Hirano [9, Theorem 1]). *Let R be a* 2-*primal ring. Then the following statements are equivalent*:

(a) *R is strongly π-regular.*
(b) *R is π-regular.*
(c) *$R/J(R)$ is π-regular and $J(R)$ is nil.*
(d) *Every prime ideal of R is a maximal one-sided ideal.*

The following example shows that in a 2-primal ring "weak π-regularity" is not the same "π-regularity". Hence in Proposition 3, condition "(b)" cannot be replaced by the condition "R is right weakly π-regular".

Example 4. Let D be a simple domain which is not a division ring. We consider the ring

$$R=\left\{\begin{pmatrix} a & b\\ 0 & a\end{pmatrix}\,\middle|\, a,b\in D\right\}.$$

Then

(1) it can be easily checked that R is 2-primal with

$$P(R)=\begin{pmatrix} 0 & D \\ 0 & 0 \end{pmatrix}.$$

(2) R is right weakly π-regular: If $x\in P(R)$, then $x^2=0$ and hence $x^2\in x^2Rx^2R$. Suppose $x\notin P(R)$. Then since $R/P(R)\cong D$ is simple, $RxR+P(R)=R$. Thus $\alpha+\beta=1$ for some $\alpha\in RxR$ and $\beta\in P(R)$. But $\beta^2=0$ and so $1=(\alpha+\beta)^2\in RxR$. Hence $x\in xRxR$. Therefore R is right weakly π-regular.

(3) R is not π-regular: Let $a\neq 0$ and 1. Then

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}^n \notin \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}^n R \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}^n$$

for any positive integer n.

As a parallel result to this Proposition 3, we obtain the following result.

Proposition 5. *Let R be a 2-primal ring. Then the following statements are equivalent*:

(a) *$R/J(R)$ is right weakly π-regular and $J(R)$ is nil.*

(b) *Every prime ideal of R is maximal.*

Proof. (a)$\Rightarrow$(b): Assume (a). Since $J(R)$ is nil and R is 2-primal, $J(R)=P(R)$. By [3, Corollary 9], every prime ideal of R is maximal.

(b)$\Rightarrow$(a): Assume (b). By [3, Corollary 9], $R/P(R)$ is right weakly π-regular and so $R/J(R)$ is right weakly π-regular. Now let $a\in J(R)$. Consider $\overline{a}\in\overline{R}=R/P(R)$. Since $\overline{R}$ is right weakly π-regular, there exists a positive integer n such that $\overline{a}^n=\overline{a}^n\overline{r}$ for some $\overline{r}\in\overline{R}\overline{a}^n\overline{R}\subseteq\overline{J}(R)$, where $\overline{J}(R)=J(R)/P(R)$. Then $\overline{a}^n(\overline{1}-\overline{r})=\overline{0}$ and so $a^n\in P(R)$. Since R is 2-primal, $a\in P(R)$ and hence $J(R)$ is nil. $\square$

Related to this Proposition 5, we noted that there exists a 2-primal ring whose prime ideals are maximal but neither right nor left weakly π-regular [3, Example 12].

However, we have the following theorem.

Theorem 6. *Let R be a 2-primal ring whose primitive factor rings are Artinian. Then the following statements are equivalent*:

(a) *R is strongly π-regular.*

(b) *R is π-regular.*

(c) *R is weakly π-regular.*

(d) *R is right weakly π-regular.*

(e) *$R/J(R)$ is right weakly π-regular and $J(R)$ is nil.*

(f) *Every prime ideal of R is maximal.*

Proof. It suffices to show that (f) $\Rightarrow$ (a). Assume (f). Let P be a prime ideal of R. Then P is maximal, so it is primitive. Thus R/P is Artinian. Hence R/P is strongly π-regular and so R is strongly π-regular by [7, Theorem 2.1]. $\square$

Recall that a ring R is said to be *right* (*left*) *quasi-duo* if every maximal right (left) ideal of R is two sided. A ring R is said to be *of bounded index* (*of nilpotency*) if there exists a positive integer n such that $a^n = 0$ for all nilpotent elements a of R.

The following theorem extends [4, Proposition 2.14].

Theorem 7. *Let R be a right quasi-duo ring. Then the following statements are equivalent*:

(a) *R is right weakly π-regular.*

(b) *R is strongly π-regular.*

Proof. It is enough to show that (a) $\Rightarrow$ (b). Assume (a). Then for any $a \in R$ there exists a positive integer n such that $a^nR = a^nRa^nR$. We claim that $aR + r(a^n) = R$, where $r(a^n)$ is the right annihilator of a^n. If not, there is a maximal right ideal M of R such that $aR + r(a^n) \subseteq M$. Then $a^nR = a^nM$, and so $a^n = a^nb$ for some $b \in M$. Hence $a^n(1 - b) = 0$ and so $1 - b \in r(a^n) \subseteq M$, which is a contradiction. Therefore R is strongly π-regular. $\square$

The fact that a right duo ring (i.e., a ring whose right ideals are two sided) implies 2-primal lead one to conjecture that a right quasi-duo ring is 2-primal. However in the following examples, the rings R are right quasi-duo but not 2-primal. Moreover, these examples show that the conditions of the following Proposition 12 and Corollary 13 are required.

Example 8. Let T be the $n \times n$ upper triangular matrix ring over a field F, where n is an infinite cardinal number. Then by [16, Proposition 2.1] T is right quasi-duo. Now consider $R = T[[x]]$, where $T[[x]]$ denotes the ring of formal power series over T. Let M' be a maximal right ideal of R. Note that M' is generated by x and a maximal right ideal M of T. Since T is right quasi-duo, M' is a two-sided ideal of R. Thus R is also right quasi-duo.

Next, we claim that R is not 2-primal. Let e_{ij} be the infinite matrix over F with (i, j)-entry the identity 1 and elsewhere the zero element 0. Now in the formal power series ring R, let

$$f(x) = e_{12}x + (e_{34} + e_{56})x^2 + \cdots + \left(\sum_{i=0}^{2^{n-1}-1} e_{(2^n+2i-1)(2^n+2i)}\right) x^n + \cdots$$

and

$$g(x) = e_{23}x + (e_{45} + e_{67})x^2 + \cdots + \left(\sum_{i=0}^{2^{n-1}-1} e_{(2^n+2i)(2^n+2i+1)}\right) x^n + \cdots .$$

Then $f(x)^2 = 0$ and $g(x)^2 = 0$. But

$$f(x) + g(x) = (e_{12} + e_{23})x + (e_{34} + e_{45} + e_{56} + e_{67})x^2 + \cdots$$
$$+ \left(\sum_{i=0}^{2^n-1} (e_{(2^n+2i-1)(2^n+2i)} + e_{(2^n+2i)(2^n+2i+1)}) \right) x^n + \cdots$$

and by setting

$$\alpha_n = \sum_{i=0}^{2^n-1} (e_{(2^n+2i-1)(2^n+2i)} + e_{(2^n+2i)(2^n+2i+1)}),$$

we have $\alpha_n^{2^n-1} \neq 0$ and $\alpha_n^{2^n} = 0$. Thus $f(x) + g(x)$ is not nilpotent; whence $f(x) \notin P(R)$ or $g(x) \notin P(R)$. Therefore R is not 2-primal. Moreover R is of no bounded index and $J(R)$ is not nil.

Example 9. Let F be a field and S denote the full ring of 2×2 matrices over F and

$$T = \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \in S \;\middle|\; a \in F \right\}.$$

We consider the ring $R = T + xS[[x]]$, where $S[[x]]$ denotes the ring of formal power series over S. Then $xS[[x]]$ is the unique maximal left and right ideal of R. Thus R is a right quasi-duo ring and $J(R) = xS[[x]]$ is not nil. Moreover, R is a semiprime PI-ring of bounded index 2. But R is not 2-primal, since

$$x \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

is a nonzero nilpotent element.

Example 10 (*Birkenmeier et al.* [4, *Example* 3.3])**.** Let G be an abelian group which is the direct sum of a countably infinite number of infinite cyclic groups; and denote by $\{b(0), b(1), b(-1), \ldots, b(i), b(-i), \ldots\}$ of basis of G. Then there exists one and only one homomorphism $u(i)$ of G, for $i = 1, 2, \ldots$ such that $u(i)(b(j)) = 0$ if $j \equiv 0 \pmod{2^i}$ and $u(i)(b(j)) = b(j-1)$ if $j \not\equiv 0 \pmod{2^i}$. Denote U the ring of endomorphisms of G generated by the endomorphisms $u(1), u(2), \ldots$. Now, let A be the ring obtained from U by adjoining the identity map of G. Let $= A \otimes_{\mathbb{Z}} \mathbb{Q}$, where $\mathbb{Z}$ is the ring of integers and $\mathbb{Q}$ is the field of rationals. Then R is a right quasi-duo ring of no bounded index and $J(R)$ is nil but R is not 2-primal. R is also a semiprime strongly π-regular ring, but there exists a prime ideal of R which is not maximal. If every prime ideal of R is maximal, then $J(R) = P(R) = 0$, which is a contradiction.

Now, we show that a right quasi-duo ring R of bounded index with $J(R)$ nil is 2-primal.

Lemma 11. *Let I be a right ideal of R and n a positive integer. If $a^n = 0$ for all $a \in I$, then $a^{n-1}Ra^{n-1} = 0$.*

Proof. For any $r \in R$, let $b = a^{n-1}r \in I$. Then $ab = a(a^{n-1}r) = a^n r = 0$. Thus $(b+a)^n = b^n + b^{n-1}a + b^{n-2}a^2 + \cdots + ba^{n-1} + a^n$. Since $(b+a)^n = 0$ and $a^n = 0 = b^n$, then we have

$$\begin{aligned} 0 &= b^{n-1}a + b^{n-2}a^2 + \cdots + b^2a^{n-2} + ba^{n-1} \\ &= b(a^{n-1}r)b^{n-3}a + b(a^{n-1}r)b^{n-4}a^2 + \cdots + b(a^{n-1}r)a^{n-2} + ba^{n-1} \\ &= ba^{n-1}(sa+1) \quad \text{for some } s \in R. \end{aligned}$$

Since $(sa)^{n+1} = s(as)^n a = 0$, $sa + 1$ is invertible. Thus $ba^{n-1} = 0$ and hence $a^{n-1}ra^{n-1} = 0$. Therefore $a^{n-1}Ra^{n-1} = 0$. $\square$

Proposition 12. *Let R be a right quasi-duo ring of bounded index with $J(R)$ nil. Then R is a 2-primal ring.*

Proof. Since R is right quasi-duo and $J(R)$ is nil, $N(R) = J(R)$ by [16, Lemma 2.3]. Let m be the bounded index of R. If $m = 1$, then $P(R) = N(R) = 0$. Let $m \geq 2$ and $J(R) \neq 0$. Then $J(R) = P(R)$. If not, then $J(R)/P(R)$ is a nonzero nil ideal of $\overline{R} = R/P(R)$ with bounded index $n \geq 2$. Thus there exists a nonzero $\overline{a} \in J(R)/P(R)$ such that $\overline{a}^{n-1} \neq \overline{0}$. By Lemma 11, $\overline{a}^{n-1}\overline{R}\overline{a}^{n-1} = \overline{0}$ and so $(\overline{R}\overline{a}^{n-1}\overline{R})^2 = \overline{0}$. Since $\overline{R}$ is semiprime, $\overline{R}\overline{a}^{n-1}\overline{R} = \overline{0}$, which is a contradiction. Therefore R is 2-primal. $\square$

Note that in Proposition 12 the conditions "R is of bounded index" and "$J(R)$ is nil" are not superfluous (Examples 9 and 10).

Corollary 13. *Let R be a right quasi-duo ring of bounded index. Then the following statements are equivalent*:

(a) *R is strongly π-regular.*
(b) *R is π-regular.*
(c) *$R/J(R)$ is π-regular and $J(R)$ is nil.*
(d) *R is weakly π-regular.*
(e) *R is right weakly π-regular.*
(f) *$R/J(R)$ is right weakly π-regular and $J(R)$ is nil.*
(g) *$R/P(R)$ is right weakly π-regular.*
(h) *Every prime ideal of R is maximal.*

Proof. (a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (d) $\Leftrightarrow$ (e) and (c) $\Leftrightarrow$ (f): These follow from Theorem 7.

(e) $\Rightarrow$ (f) and (e) $\Rightarrow$ (g): These are obvious.

(g) ⇒ (a): Assume (g). Then $R/P(R)$ is strongly π-regular by Theorem 7. Thus R is strongly π-regular by [7, Theorem 2.1].

(f) ⇒ (h): Assume (f). Then by Proposition 12 and Proposition 5, every prime ideal of R is maximal.

(h) ⇒ (a): It follows from [16, Theorem 2.5]. □

In Corollary 13, the condition "R is of bounded index" is not superfluous (Example 10).

Remark. In Example 4, the ring R is 2-primal right weakly π-regular which is of bounded index 2. But it is not π-regular. The ring in [3, Example 12] is a 2-primal ring whose prime ideals are maximal, and it is of bounded index 2. But it is neither right nor left weakly π-regular.

Recall that a ring R is said to be *weakly right duo* if for every $a \in R$ there is a positive integer $n = n(a)$, depending on a, implies $a^n R$ is a two-sided ideal of R [15]. A ring R is said to be *abelian* if every idempotent element of R is central. Note that every weakly right duo ring is abelian right quasi-duo [16, Proposition 2.2] but, in general, is of no bounded index. For example, the ring R in Example 10 is weakly right duo but it is of no bounded index.

However, we have the following proposition.

Proposition 14. *Let R be a weakly right duo ring. Then the following statements are equivalent*:

(a) *R is strongly π-regular.*
(b) *R is π-regular.*
(c) *R is right* (*left*) *weakly π-regular.*
(d) *$R/J(R)$ is strongly regular.*

Proof. It is sufficient to show that (c) ⇒ (d) and (d) ⇒ (a).

(c) ⇒ (d): Assume (c). Then $R/J(R)$ is strongly π-regular by Theorem 7. Also by [15, Corollary 2], $R/J(R)$ is reduced and so it is strongly regular.

(d) ⇒ (a): It follows from [15, Corollary 2; 1, Theorem 3] and Theorem 7. □

In weakly right duo rings, if every prime ideal of R is maximal then R is strongly π-regular [16, Theorem 2.5]. However, the converse is not true (Example 10).

Now we investigate the connection between the weak π-regularity and the maximality of prime ideals in PI-rings.

Theorem 15. *Let R be a prime right Goldie ring. If R is right weakly π-regular*, *then R is a simple ring.*

Proof. Let I be a nonzero two-sided ideal of R. Since R is prime, I is essential as a right ideal. Then by [5, Theorem 1.10], I contains a regular element $a \in R$. Since R is right weakly π-regular, there exists a positive integer n such that $a^nR = a^nRa^nR$ and so $a^n = a^nb$ for some $b \in Ra^nR$. Thus $a^n(1-b) = 0$. Now a^n is also regular element and so $b = 1$; whence $I = R$. Therefore R is a simple ring. □

Corollary 16. *Let R be a PI-ring. If R is right weakly π-regular, then every prime ideal of R is maximal.*

Proof. Let P be a prime ideal of R. Then R/P is a prime PI-ring. So by Posner's Theorem [12], R/P is right Goldie. By Theorem 15, R/P is simple. Hence every prime ideal of R is maximal. □

Theorem 17. *Let R be a PI-ring. Then the following statements are equivalent*:

(a) *R is right (left) weakly π-regular.*

(b) *$R/J(R)$ is right (left) weakly π-regular and $J(R)$ is nil.*

Proof. (b)$\Rightarrow$(a): Assume (b). If R is not right weakly π-regular, then R is not strongly π-regular. Thus there exists a prime ideal P of R such that R/P is not strongly π-regular by [7, Theorem 2.1]. Since $R/J(R)$ is a right weakly π-regular and PI-ring, $R/J(R)$ is strongly π-regular by [9, Theorem 4]. If $J(R) \subseteq P$, then a homomorphic image R/P of $R/J(R)$ is strongly π-regular, which is a contradiction. Thus $J(R) \nsubseteq P$ and so $(J(R)+P)/P$ is a nonzero two-sided ideal of R/P. Since R/P is a prime PI-ring, and so it is right Goldie, $(J(R)+P)/P$ contains a regular element $a+P$ with $a \in J(R)$ by [5, Theorem 1.10]. However, $J(R)$ is nil by assumption so $a^k = 0$ for some integer k; whence $(a+P)^k = 0$ in R/P, which is also a contradiction. Therefore R is right weakly π-regular. □

As a byproduct of Corollary 16 and Theorem 17, we obtain Corollary 18 which includes [7, Theorem 2.3; 9, Theorem 4].

Corollary 18. *Let R be a PI-ring. Then the following statements are equivalent*:

(a) *R is strongly π-regular.*

(b) *R is π-regular.*

(c) *$R/P(R)$ is π-regular.*

(d) *$R/J(R)$ is π-regular and $J(R)$ is nil.*

(e) *R is weakly π-regular.*

(f) *R is right (left) weakly π-regular.*

(g) *$R/J(R)$ is right weakly π-regular and $J(R)$ is nil.*

(h) *Every prime factor ring of R is right (left) weakly π-regular.*

(i) *Every prime ideal of R is maximal.*

(j) *Every prime factor ring of R is simple Artinian.*

Note that all primitive factor rings of a PI-ring and a right quasi-duo ring are Artinian.

Related to Corollaries 13 and 18, we have the following proposition.

Proposition 19. *Let R be of bounded index whose primitive factor rings are Artinian. Then the following statements are equivalent*:

(a) *R is strongly π-regular.*

(b) *R is π-regular.*

(c) *$R/P(R)$ is π-regular.*

(d) *Every prime ideal of R is maximal.*

Proof. (a)$\Rightarrow$(b)$\Rightarrow$(c): It is clear.

(c)$\Rightarrow$(d): Assume (c). Since R is of bounded index, $R/P(R)$ is also of bounded index. Then by [10, Proposition 2], every prime ideal of $R/P(R)$ is maximal. Therefore every prime ideal of R is maximal.

(d)$\Rightarrow$(a): Assume (d). Let P be a prime ideal of R. Then R/P is Artinian and so R/P is strongly π-regular. Thus by [7, Theorem 2.1], R is strongly π-regular. □

The conditions of Proposition 19(1) "R is of bounded index" and (2) "every primitive factor ring of R is Artinian" are not superfluous.

Example 20. (1) In Example 10, the ring R is a right quasi-duo ring, so every primitive factor ring of R is Artinian. R is also a strongly π-regular ring of no bounded index. But there exists a prime ideal of R which is not maximal.

(2) [3, Example 13]. Let $W = W_1[F]$ be the first Weyl algebra over a field F of characteristic zero. Now we consider the ring

$$R = \{(a_i)_{i=1}^{\infty} \mid a_i \in Mat_2(W) \text{ is eventually a constant upper triangular matrix}\},$$

where $Mat_2(W)$ denotes the full ring of 2×2 matrices over W. Then R is a semiprime ring of bounded index 2 whose prime ideals are maximal. But R is not π-regular. If every primitive factor ring of R is Artinian, then by Proposition 19, R is π-regular, which is a contradiction.

We conclude this paper with the following question.

Question 1. *If R is a right weakly π-regular ring of bounded index whose primitive factor rings are Artinian*, *then is R π-regular*?

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